Circular Motion and Gravitation Revision Guide

Centripetal force and acceleration

- Centripetal acceleration is equal to the equation below

$$a_c = \frac{v^2}{r}$$

- Centripetal force is always provided by some other force, such as friction and tension, and it is equivalent to a system's net force.

$$F_c = ma_c$$

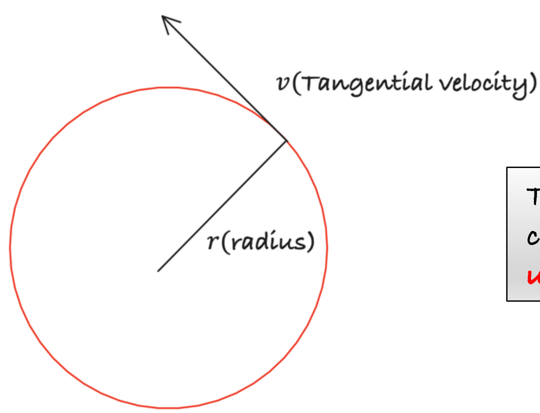
$$F_c = m\frac{v^2}{r}$$

$$a_c = \frac{v^2}{r}$$

a_c = Centripetal acceleration

Basic properties of circular motion

- The direction of centripetal force always points to the center of its circular path
- The direction and linear (tangible) velocity is perpendicular to its radius. The is the same as saying that the direction of velocity is perpendicular to its centripetal force



They are two types of circular motion: **uniform and non-**

Properties for uniform circular motion

- The direction of centripetal force always points to its center
- The direction and linear (tangible) velocity is perpendicular to its radius (Velocity is perpendicular to its centripetal force).
- Constant speed (the magnitude of tangible velocity is constant), meaning that there is no acceleration for tangible velocity
- Work done by centripetal force is zero. There are two ways for deriving this: work-energy theorem or basic trigonometry

Derivation of Work-Energy Theorem and Basic trigonometry
$\Delta W = \Delta K$ Work done is equal to change in kinetic energy $F \cdot d \cos\theta = \frac{1}{2}m(v_f^2 - v_i^2)$ $F_c \cdot d \cos\theta = \frac{1}{2}m(v_f^2 - v_i^2)$ Since velocity is constant, centripetal force is also constant, so work done by centripetal force is also zero Since the direction of centripetal force is perpendicular to its velocity, $\cos\theta = \cos 90 = 0$, so work done is equal to zero

Some other important concepts

Angular displacement

- Angular displacement refers to arch length an object travels in radians

$$L = r\theta$$

(L=angular displacement, R=radius, θ = angle in radian)

Angular velocity

Angular velocity refers to the rate of change in angular displacement. It is a vector quantity with a unit of radian per second, calculated by the equation below.

$$\omega = \frac{\theta}{\Delta t}$$

(θ = angular displacement, θ= change in time, ω= angular velocity)

We can express speed in terms of angular velocity, as shown below

$$v = \frac{L}{t} = \frac{r\theta}{t}$$

Using the formular of angular velocity, we get

$$v = r\omega$$

Note: speed means the magnitude of tangible velocity here.

Period

- Period is the time for one revolution, and it has a unit of 1/s (1 over second)

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = 2\pi f$$

Frequency

- Frequency is defined as the rate of rotation, or the number of rotations in 1 unit second. It has a unit of second, also known as Hertz, Hz
- Frequency and period are reciprocal to each other as shown by the equation below

$$T = \frac{1}{f}$$

Connecting all formula together

Circular motion maintains many formulas, so it is important to summarize and build connections. Some most frequently equations are listed below.

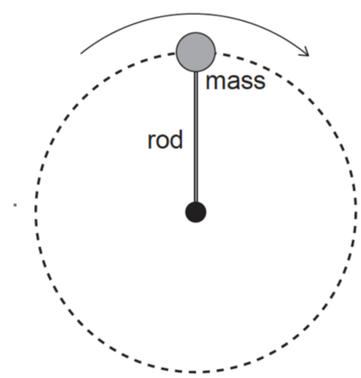
Centripetal force	$F_c = m\frac{v^2}{r}$
Angular displacement	$L = r\theta$
Angular velocity	$\omega = \frac{\theta}{\Delta t}$
Period	$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$
Tangible velocity	$v = r\omega$

Some further derivations are listed below

Centripetal force	$F_c = m\frac{(r\omega)^2}{r} = mr\omega^2$
	$F_c = m\frac{(2\pi r/T)^2}{r} = \frac{4m\pi^2 r}{T^2}$
Period	$T = \frac{2\pi r}{v}$
	$T = \frac{2\pi}{\omega}$
	$T = 2\pi f$

Some frequent testing points

Assume that you are rolling a ball in a vertical circular path. At which position is the magnitude of tension the greatest. (See problem 2). Write the equation of tension in term of centripetal force in different positions.



Position	Force Analysis	Magnitude of Tension
For object on the top	$F_c = T + mg$	$T = F_c - mg$
For object on the bottom	$F_c = T - mg$	$T = F_c + mg$
For object half-way up	$F_c = T$	$F_c = T$
Tension on the bottom is the greatest, minimum at the top		

Roller Coaster Analysis (Minimum Velocity at the topic)

For a roller coaster, its minimum velocity required is equal to $\sqrt{gR}$, where **R** is radius and **g** is gravitational acceleration.

$$F_c = N - mg$$

To get the minimum value of centripetal force, we set the normal force N equal to zero, so centripetal force is equal to gravitational force mg.

$$F_c = mg$$

Plug in the formular of centripetal force

$$m\frac{v^2}{r} = mg$$

We get the result that

$$v = \sqrt{gR}$$

> To calculate minimum velocity at the topic, always set normal force N=0

Newton's Law of Universal Gravitation

Newton's law of universal gravitation states that every point mass in the universe attract every other point mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers.

$$F_g = G\frac{Mm}{r^2}$$

(F_g = force of gravitation, M = mass of object 1, m = mass of object 2, r = distance between 2 objects)

$$F_g \propto \frac{1}{r^2}$$

Gravitational Field Strength

- Gravitational field is the force experienced by a unit mass, calculated by the equation below.

$$g = \frac{GM}{r^2}$$

G=universal gravitation constant

M= mass produced force

r = distance between M and m.

> $g = \frac{F}{m} = \frac{GMm/r^2}{m}$

- Gravitational field is used to explain the influences that a massive body produced into the space into itself
- It is a vector quantity with a unit of Newton per kilogram, N/kg

Circular Motion and Gravitation Connection (Orbital Motion)

Orbital Speed

- Orbital velocity is the speed required to achieve orbit around a celestial body, such as a planet or a star, calculated by the below equation

$$v = \sqrt{\frac{GM}{r^2}}$$

- This is because for a planet orbiting around another object in space, its centripetal force is equal to the gravitational attraction it received, given us the below equation

$$m\frac{v^2}{r} = \frac{GMm}{r^2}$$

Solving for velocity v, we obtain the above equation.

Gravitational Potential Energy

- When an object is near the surface of a planet, its gravitational potential energy is approximately equal to $U_g = mgh$. However, for objects in space, a more accurate equation is needed, shown by the equation below.

$$g = \frac{F}{m} = \frac{GMm/r^2}{m}$$

$$U_g = -\frac{GMm}{r}$$

Conservation of Energy

- The total energy in a system is conserved if there is no external force acting on it

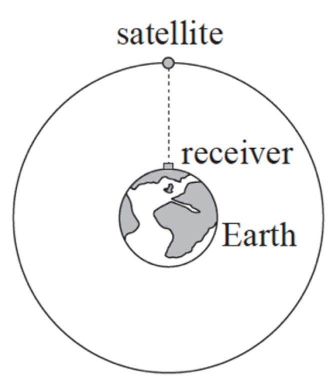
$KE_i + PE_i = KE_f + PE_f$	
When object is near the surface of a planet	When object is in space
$\frac{1}{2}mv_i^2 + mgr_i = \frac{1}{2}mv_f^2 + mgr_f$ $\frac{1}{2}m(v_f^2 - v_i^2) = mg(r_i - r_f)$	$\frac{GMm}{2r_i} - \frac{GMm}{r_i} = \frac{GMm}{2r_f} - \frac{GMm}{r_f}$ $\frac{GMm}{2}\left(\frac{1}{r_i} - \frac{1}{r_f}\right) = GMm\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$

One Classical Types of Problem

Calculating Period

Calculating period of the satellite given the mass of earth, radius between two objects. For this kind of problem, use the below equation

$$T = \sqrt{\frac{4\pi^2 r^3}{M}}$$



This is because

This is an important formular that's frequently tested.

$$G\frac{Mm}{r^2} = m\frac{v^2}{r}$$

$$F_c = \frac{(2\pi r/T)^2}{r} = m\frac{4\pi^2 r}{T^2}$$

$$G\frac{Mm}{r^2} = m\frac{4\pi^2 r}{T^2}$$

Revision Questions (Paper 1)

1. A particle of mass *m* is moving with constant speed *v* in uniform circular motion. What is the total work done by the centripetal force during one revolution?
   A. Zero
   B. $\frac{mv^2}{2}$
   C. mv^2
   D. $2\pi mv^2$

2. An object at the end of a wooden rod rotates in a vertical circle at a constant angular velocity. What is correct about the tension in the rod?

   A. It is greatest when the object is at the bottom of the circle.
   B. It is greatest when the object is halfway up the circle.
   C. It is greatest when the object is at the top of the circle.
   D. It is unchanged throughout the motion.

3. Two satellites of mass *m* and 2*m* orbit a planet at the same orbit radius. If *F* is the force exerted on the satellite of mass *m* by the planet and a is the centripetal acceleration of this satellite, what is the force and acceleration of the satellite with mass 2*m*?

	Force	Acceleration
A.	$2F$	a
B.	$2F$	$\frac{a}{2}$
C.	F	a
D.	F	$\frac{a}{2}$

ANS: A

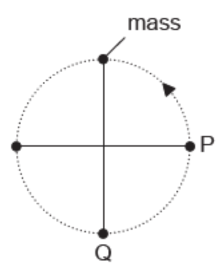
4. What is the correct definition of gravitational field strength?
   A. The mass per unit weight
   B. The weight of a small test mass
   C. The force acting on a small test mass
   D. The force per unit mass acting on a small test mass

ANS: D

5. A spherical planet of uniform density has three times the mass of the Earth and twice the average radius. The magnitude of the gravitational field strength at the surface of the Earth is g. What is the gravitational field strength at the surface of the planet?
   A. $6g$
   B. $\frac{2}{3}g$
   C. $\frac{3}{4}g$
   D. $\frac{3}{2}g$

ANS: C

6. A mass attached to a string rotates in a gravitational field with a constant period in a vertical plane.



How do the tension in the string and the kinetic energy of the mass compare at P and Q?

	Tension in the string	Kinetic energy of mass
A.	greater at P than Q	greater at Q than P
B.	greater at Q than P	greater at Q than P
C.	greater at P than Q	same at Q and P
D.	greater at Q than P	same at Q and P

ANS: B

An object of mass *m* at the end of a string of length *r* moves in a vertical circle at a constant angular speed ω.

What is the tension in the string when the object is at the bottom of the circle?

A. $m(\omega^2 r + g)$

B. $m(\omega^2 r - g)$

C. $mg(\omega^2 r + 1)$

D. $mg(\omega^2 r - 1)$